

On the origin of clustering of frequency ratios in the atoll source 4U 1636-53

Gabriel Török¹, Marek A. Abramowicz^{1,2,3}, Pavel Bakala¹, Michal Bursa⁴, Jiří Horák⁴, Paola Rebusco⁵, Zdeněk Stuchlík¹

¹ Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, 746-01 Opava, CZ

² Department of Physics, Göteborg University, S-412 96 Göteborg, SE

³ Copernicus Astronomical Centre PAN, Bartycka 18, 00-716 Warsaw, PL

⁴ Astronomical Institute of the Academy of Sciences, Boční II 1401/1a, 141-31 Praha 4, CZ

⁵ MIT Kavli Institute for Astrophysics and Space Research, 77 Massachusetts Avenue, 37, Cambridge, MA 02139, US

e-mail: terek@volny.cz, marek.abramowicz@physics.gu.se, pavel.bakala@fpf.slu.cz, bursa@astro.cas.cz, horak@astro.cas.cz, pao@space.mit.edu, zdenek.stuchlik@fpf.slu.cz

Abstract. A long discussion has been devoted to the issue of clustering of the kHz QPO frequency ratios in the neutron star sources. While the distribution of ratios inferred from an occurrence of a single QPO seems to be consistent with a random walk, the distribution based on simultaneous detections of both peaks indicates a preference of ratios of small integers. Based on the public RXTE data we further investigate this issue for the source 4U 1636–53. Quality factors and rms amplitudes of both the QPOs nearly equal to the points where the frequencies are commensurable, and where the twin QPO detections cluster. We discuss a connection of the clustering with the varying properties of the two QPO modes. Assuming approximative relations for the observed correlations of the QPO properties, we attempt to reproduce the frequency and ratio distributions using a simple model of a random-walk evolution along the observed frequency-frequency correlation. We obtain results which are in qualitative agreement with the observed distributions.

Keywords: X-rays: binaries — Stars: neutron — Accretion, accretion disks

1 Introduction

Since the paper of Abramowicz et al. (2003), the issue of distribution of kHz QPOs in neutron-star low mass X-ray binaries has been discussed extensively. In their work, Abramowicz et al. examined simultaneous detections of the upper and lower QPOs in the Z-source Sco X-1. The authors show that that the ratios of the lower and upper QPO frequencies cluster most often close to the value $\nu_L/\nu_U = 2/3$. They find also evidence for the second peak in a distribution of frequency ratios at $\nu_L/\nu_U \approx 0.78$. This value is remarkably close to another ratio of small integers, $4/5 = 0.8$. In the most recent paper, Török et. al (2008) have examined occurrences of the twin QPOs in the atoll source 4U 1636–53 applying the same methodology as Abramowicz et al. (2003). They find that the distribution of the (inverse) frequency ratios ν_U/ν_L of two simultaneously detected QPOs peaks near $3/2$ and $5/4$.

A preference of the commensurable frequency ratios in kHz QPO data of various sources has been systematically checked by a group of Belloni and his collaborators. Belloni et al. (2005) have re-examined the ratio distribution in Sco X-1 and later also in a larger sample comprising four atoll sources including 4U 1636–53 (Belloni et al. 2005). They argue that such clustering does not provide any useful information because frequencies of the two QPOs are correlated and the distribution of the ratio of two correlated quantities is completely determined by the distribution of one of them.

Keeping this argument, a recent study of Belloni et al. (2007) based on a systematic long term observation of 4U 1636–53 concludes that there is no preferred frequency ratio.

The aparent disagreement in conclusions of the two groups comes from a confusion between the observed frequency distribution (the one which can be recovered from observed data) and the intrinsic distribution (the “invisible” one really produced by the source). While Abramowicz et al. (2003) and Török et. al (2008) have examined frequency ratios of the actually observed QPO pairs (twin peaks) only, the analysis of Belloni et al. (2005, 2007) study primarily distributions of frequencies of a single QPO and make implications for the distribution of the other, often invisible, QPO from the empirical correlation between frequencies.

In this paper we show that the observed distributions are affected by the way the signal from a source is being detected and analyzed. We show that the observed clustering can be understood in terms of rms amplitude and quality factor correlations with QPO frequency. Taking these correlations into account, we simulate the ratio distribution using a random walk model of QPO frequency evolution and we find that results of the simulation agree with empirical data.

2 Properties of oscillation modes on large frequency range

In the process of data reduction and searching for QPOs, an important quantity is the significance S of the peak in PDS, which measures the peak prominence. Shape of a peak in the PDS is most often fitted by a Lorentzian. Usually, $S \geq 2-4$ is being used as the low threshold limit for detections and only peaks that have their significances greater than this limit are considered as QPOs. Thus, this imposes a certain selection criterion, which could consequently affect the distribution of detections.

The significance S is given by the relation between the integral area of a Lorentzian in PDS and its error. For a particular detection, it depends on observational conditions, on the quality factor Q of the peak (defined as the QPO centroid frequency over the peak full-width at its half-maximum) and on the fractional root-mean-squared amplitude r (a measure for the signal amplitude given as a fraction of the total source flux that is proportional to the root mean square of the peak power contribution to the total power spectrum), $S = k r^2 \sqrt{Q/\nu}$, where the time-varying factor $k(t) = I(t) \sqrt{T}$ depends on the total length of observation T and the instantaneous source intensity I , which at a given time same for both upper and lower peak¹.

Barret et al. (2005a,b,c, 2006) have shown that both quality factors and rms amplitudes are determined by frequency and moreover that their profiles greatly differ between lower and upper QPO modes. The quality factor of the upper QPO is usually small and tends to stay at an almost constant level around $Q_u \sim 10$. In contrast, the lower QPO quality factor improves with frequency and can reach up to $Q_l \sim 200$ before a sharp drop of coherence at high frequencies. Amplitudes of upper QPOs generally decrease with frequency, while the lower QPO amplitudes show first an increase and then they start to decay too.

¹The standard process of the QPO determining is in detail described in van der Klis (1989).

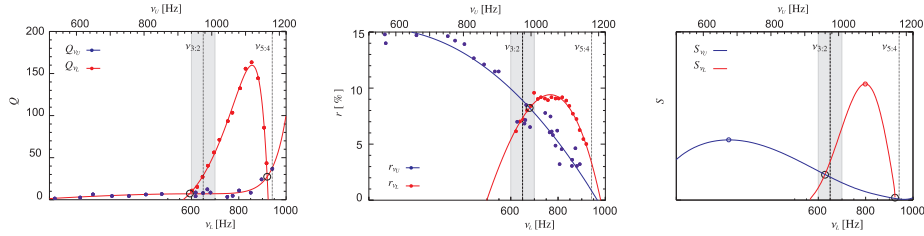


Figure 1: The quality factor (*left*), rms amplitude (*middle*) and inferred significance (*right*) behaviour in atoll source 4U 1636–53. Red points represent lower QPO data, blue points are for upper QPO data. Data in first two panels comes from the study of Barret et al. (2005b) and cover large range of frequencies available via shift-add method through all segments of RXTE observations. Continuous curves are obtained from interpolation by several exponentials (see, e.g., Török 2007). The prospected course of the QPO significance in the right panel is determined by the rms amplitude and quality factor profiles ($S \propto r^2 \sqrt{Q/\nu}$). Frequency axes are related using frequency correlation ($\nu_U = 0.701\nu_L + 520\text{Hz}$; Abramowicz et al. 2005).

Figure 1 shows the behaviour of amplitudes and quality factors of individual QPO modes in 4U 1636–53 and how they change with frequencies.² The displayed data of Barret et al. (2005b) cover large frequency range available through the shift-add technique over all RXTE observations (see Méndez et al. 1998, 1999; Barret et al. 2005a,b,c, for details).

Note that both of the two properties are becoming similar as the frequency approaches points corresponding to 3/2 or 5/4 ratio (the equality of amplitudes have been reported by Török 2007). In the right panel of Figure 1 we then plot the significances of the two oscillation modes, inferred from the combination on the two plots, while we keep the intensity I and observing time t constant (for simplicity). It is clearly visible that there is a similar equality of QPO significances close to points, where the frequencies are close to the 3/2 or 5/4 ratio (as a result of comparable Q and r at those points), while they are much different elsewhere. We will hereafter call the points of equal significances as the “3/2” and “5/4” points. We may also observe that the upper QPO mode is usually strong (much more significant) left from the 3/2 point (at lower frequencies), while right from 3/2 the lower QPO mode dominates.

3 Clustering of frequency ratios

It is likely that if QPOs are produced in a source, they are always produced in pairs. Because the strength of oscillations is usually around the sensitivity threshold of measurements, often only one (the stronger) QPO is detected. Around the special points 3/2 and 5/4, where significances are comparable, there is a good chance that if one mode can be detected the other could be detected as well, because both peaks have nearly the same properties. Indeed, this agrees with what is observed and has been laboured or challenged many times (Abramowicz et al. 2003; Belloni et al. 2005; Bulik 2005; Yin & Zhao 2007; Belloni et al. 2007) that pairs of QPOs cluster close to the 3/2

²In the figure we use a correlation $\nu_U = 0.701\nu_L + 520\text{Hz}$ from Abramowicz et al. 2005 (see also Belloni et al. 2005; Zhang et al. 2006).

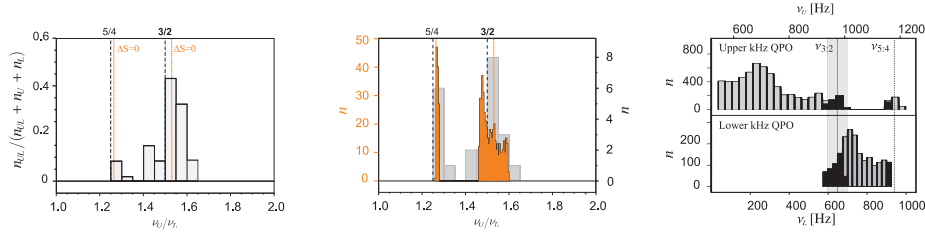


Figure 2: The distribution of observed frequency ratios. *Left*: The fraction of the number of observations with simultaneous detections n_{UL} to the number of observations in which at least one QPO has been detected $n_{UL} + n_U + n_L$ (where n_U and n_L are respectively the numbers of observations with detections of the upper or lower QPO only). *Middle*: Simulated ratio distribution assuming a random-walk in frequency and variable count rate (see text). The gray underlying histogram in the first two panels shows the actual observed ratio distribution of twin QPO peaks (the data in both panels are those discussed in Török et. al 2008). *Right*: The individual distributions of lower and upper QPO frequencies from the random walk simulation. Black-shaded portions of bars represent simultaneous occurrences of both modes (twin QPOs) as shown in the middle panel.

and some other small rational number ratios.

From time to time, the conditions at the source become such that both QPOs can be detected simultaneously regardless of their frequency, only because of their actual high brightness (as the observational sensitivity is relatively low). These events allows us not only to see QPO pairs close to the critical points, but sporadically also all the way along the frequency-frequency correlation line, even far from $3/2$.

The clustering of frequency ratios close to $3/2$ is in this view significantly affected by the behaviour of rms amplitudes and quality factors and namely by the fact that these quantities become equal close to that frequency ratio. This is demonstrated in Figure 2 (left), where we show a fraction of number of observations, in which both QPOs have been detected simultaneously, to a number of those, in which at least one QPO has been detected. The figure is based on data used in Török et. al (2008). Clearly, the positions of maxima remarkably well correlate with points, where the two significances equal. Moreover, these positions coincide with peaks in the distribution of frequency ratios found in Török et. al (2008) which justify a hypothesis that there is a link between QPO properties and the ratio clustering.

4 Random walk distribution model

As previously noticed in several works and further suggested by Belloni et al. (2005), the observed time evolution of QPO frequency appears consistent with a series of random walks.

This has been later criticised by Bulik (2005) who pointed out that contrary to the distributions of QPOs that appear qualitatively similar at different times, distributions arising from random walk differ significantly among different realisations (with different seeds). Nevertheless, using a simple model of random walk we attempt to at least roughly reproduce the frequency ratio distribution.

Starting with $\nu_L = 700\text{Hz}$, we model a long-term evolution of QPO frequencies

over 10000 consequent segments. Each segment consists of 50 steps, where one step is assumed to represent 32 seconds of a real observation. An independent random variation of $\pm 2\text{Hz}$ in ν_L is assigned to each step. This setup roughly corresponds to the documented frequency drifting through 32 sec integration intervals (Barret et al. 2004; Paltani et al. 2004) and each segment then mimic 1.6 kiloseconds of QPO evolution.

The QPO frequencies are averaged over each segment, and the linear correlation $\nu_U = 0.701\nu_L + 520\text{Hz}$ is considered, so that finally we obtain 10000 frequency pairs. To start with, we assume constant observational conditions (i.e. count rate and observing time), adjusting $k=1$. For each point we calculate its significance based on observed profiles of Q and rms , which are based merely on datapoints corresponding to twin peak QPO observations. Only such points are considered in the simulation, where both upper and lower QPOs have significance above 3σ level. The resulting histogram of frequency ratios shows strong clustering around 3:2 ratio, however, it does not reproduce the second peak around 5/4, which indicates that the assumption of constant count rate may not be sufficient.

As a second step, we adopt an additional (still very simplifying) assumption to the simulation that countrate is varying with frequency. The motivation here comes from a known fact that for a given source there is not a global correlation between source luminosity and QPO frequency, but the two quantities stay correlated during individual (temporary) observational events (so-called parallel-track phenomenon, e.g., Méndez et al. 1999). In the case of 4U 1636–53, the maximal countrates related to the highest observed lower QPO frequencies (up to 950Hz) are 2–3 times higher than the highest countrates at $\nu_L \sim 500\text{--}700\text{Hz}$ (see Figure 2 in Barret et al. 2005b). Thus, we keep countrate constant up to $\nu_L \sim 700\text{Hz}$ and then it is linearly increased with frequency, being about 2.5 times higher at $\nu_L \sim 950\text{Hz}$ than at $\sim 500\text{--}700\text{Hz}$.

In the middle panel of Figure 2, we show first the histogram of simultaneous occurrences of both QPO modes from our simulation on the background of the observed distribution, and in the right panel of the same figure we plot also individual simulated distributions of lower and upper QPOs. Focusing on twin QPO occurrences, we have a broad peak around 3/2 and also we obtain a more narrow peak near 5/4. While the presence of the 3/2 clustering seem to be very solid and can be reproduced with any setup, the second 5/4 peak is more subtle feature and depends much on assumed behaviour of countrate. In a real observation, its presence would apparently rely on actual source conditions (and how they would change during the observation) as well as on how the consequent analysis is done. For instance the data examined in Belloni et al. (2007) does not exhibit QPO detections above $\nu_U \sim 1000\text{Hz}$ while the data used in Török et. al (2008) does. Similarly, if we put e.g. more stiff limit on significance or consider lower countrates, we would loose the 5/4 peak.

5 Conclusions

Focused on the atoll source 4U 1636-53 we demonstrate that at frequencies, where the both QPO modes have comparable properties, there is a high probability of detecting both peaks of a twin pair simultaneously. We have found a precise match comparing the observed twin QPO distribution with our simulation based on the observed correlations

between QPO frequencies and their properties. The simulation not only reproduces the observed clustering, but it also shows the “complementarity” between upper and lower QPO distributions that has been noticed by Török et. al (2008). This suggests that the ratio clustering may origin in the exchange of dominance between the two modes when one mode fades in and the other one fades out.

Even if the intrinsic distributions of both the mode frequencies were uniform, there would be a non-trivial profile of the observed distributions and clustering of the twin peak detections around certain points (narrow regions) prominent due to behaviour of the QPO amplitudes and coherence times determined by the QPO mechanism. It will require a further detailed analysis to investigate whether the above influence of the QPO properties can explain the ratio clustering observed in 4U 1636-53 completely. For a further understanding of the ratio clustering mechanism (and importance) it is also highly needed to perform a similar analysis for the other sources. For instance, a very recent study of the atoll source 4U 1820-30 (Barret & Boutelier 2008) found that a point close to the $4/3$ value, where the ratio distribution clusters in that source, and where the amplitudes and quality factors are comparable, is most likely prominent in the intrinsic distribution³.

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³Note also that in contrary to the case of 4U 1636 they reported a lack of the twin QPO detections close to the $3/2$ value, while the amplitudes and quality factors are comparable there as well.

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